

DIAMETER OF THE COMMUTING GRAPH OF $\mathbb{R}^{n \times n}$

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ABSTRACT. The vertices of *commuting graph* of $\mathbb{R}^{n \times n}$ are non-scalar matrices; the edges are defined as pairs (u, v) satisfying $uv = vu$. One can see [1] that, for $n \geq 3$, the diameter of this graph is at least four; we give a short proof that it is exactly four. A longer proof of this result has been known [2, 3].

If the eigenvalues of a are simple, then \mathbb{R}^n is a direct sum of invariant subspaces each of which has dimension either 1 or 2; in this case, a commutes with a nontrivial idempotent. The distance between a pair of idempotents is at most 2 by Theorem 11 of [1], so that, for any matrices a_0 and a_4 , there are sequences $a_0(t), \dots, a_4(t)$ of non-scalar matrices such that $a_0(t)$ and $a_4(t)$ converge to a_0 and a_4 , respectively, and $a_{j-1}(t)$ commutes with $a_j(t)$. Multiplying $a_i(t)$ by a constant and adding a scalar matrix, we can get a singular matrix $\alpha_i(t)$ with unit norm. Since the unit sphere is compact, every $\alpha_i(t)$ is dense in some α_i , which is non-scalar and real up to adding a scalar matrix. Passing to the limit, we see that α_{j-1} and α_j commute.

REFERENCES

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